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Greybody factors and charges in Kerr/CFT

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Abstract: We compute greybody factors for near extreme Kerr black holes in $D = 4$ and $D = 5$. In $D = 4$ we include four charges so that our solutions can be continuously deformed to the BPS limit. In $D = 5$ we include two independent angular momenta so Left-Right symmetry is incorporated. We discuss the CFT interpretation of our emission amplitudes, including the overall frequency dependence and the dependence on all black hole parameters. We find that all additional parameters can be incorporated in Kerr/CFT, with central charge independent of U(1) charges.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence

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1 Introduction

The Hawking temperature of a black hole vanishes in the extreme limit $T_H \to 0$. It is therefore natural to interpret extreme black holes as ground states of the corresponding quantum theory, and so they are presumably the simplest starting point for the analysis of more general black holes with finite temperature. There are actually several inequivalent extreme limits. Writing the (inverse) Hawking temperature as

$$\frac{1}{T_H} = \frac{1}{2}\left(\frac{1}{T_R} + \frac{1}{T_L}\right), \tag{1.1}$$

we can take $T_H \to 0$ as:

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a) **BPS**: $T_R \to 0$ *with supersymmetry preserved in the limit*. These are the BPS black holes, the examples most analyzed in string theory (some reviews are [1–3]).

b) **non-BPS**: $T_L \to 0$. This is the alternative extremal limit that breaks supersymmetry completely. It is the "non-BPS branch" that has been developed recently (including [4–7]).

   Recently there has been much progress on the description of extreme Kerr black holes (including [8–10]). One of the motivations for this particular extreme limit is that many astrophysical black holes naturally spin up in the process of accretion, and so tend to approach the extreme Kerr limit. The string theory description of extreme Kerr could therefore be relevant for observations [8, 11]. Importantly, the extreme Kerr limit defines a class distinct from those above:

c) **Extreme Kerr**: $T_R \to 0$ *with supersymmetry broken in the limit*, due to the presence of angular momentum.

The BPS black hole and the extreme Kerr black hole both correspond to a definite state in the R-sector, as far as classical considerations are concerned. The difference is that the BPS black holes represent the true ground states, while the extreme Kerr black holes correspond to states that have a condensate of angular momentum carriers (see eg.[12]). The condensate breaks supersymmetry and carries a macroscopic angular momentum; but it does not carry any macroscopic entropy and so $T_R \to 0$ just as in the true ground state describing BPS black holes.

An illuminating way to analyze the various limits is to compute the frequency dependent absorption cross-section of the black holes or, equivalently (due to detailed balance), the spectrum of Hawking emission [13–18].

This means solving the (massless) Klein-Gordon equation for a scalar field in the black hole background. Despite the generality of our setting, the equation takes a strikingly simple form: it comprises some "asymptotic" terms and some "near horizon" terms. In *all* cases where the two groups of terms can be taken into account sequentially, the full solution takes the same form as the two-point correlator in a 2D CFT.

One situation where the matching procedure is justified is for near extremal black holes where the two thermal scales $T_{L,R}$ establish a hierarchy. Significantly, the Left/Right structure described above shows that *the CFT underlying Kerr can be related by continuous deformation to the BPS black holes* that are well understood. All that is needed is that one must maintain the hierarchy $T_R \ll T_L$ as the angular momentum is turned off by tuning charges. This situation makes it interesting to keep all the charges as the Kerr limit is approached. This is one of the gaps in the literature that we fill with this paper.

The discussion so far was for 4D but there is a similar story for 5D black holes. In 5D the R and L sectors are isomorphic, so there is no analogue of the non-BPS branch b). However, the relation between the BPS and the Kerr branches remains the same. Moreover, since there are two angular momenta $J_{L,R}$, the near extreme limit defined by large J_R generally leaves J_L free. In this paper we take the dependence of this parameter into account.

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The central charge of Kerr/CFT was determined in [8] using the method of [19–23] to determine the asymptotic symmetries. The result was later generalized to Kerr black holes with one or more charges in various dimensions [9, 24, 25]. Here, we are interested in the striking CFT interpretation of the supergravity correlation functions. Two point functions do not immediately depend on the central charge, but they do depend on the complex structure of the space that the CFT is defined on. We discuss the relevant scales for the background with general charges.

The feature of the wave equation that leads to correlation functions reminiscent of a CFT is the hypergeometric nature of the near horizon regime. The hypergeometric function is a character of $\mathrm{SL}(2,\mathbb{R})$, so one can try to interpret this structure as a remnant of a Virasoro algebra. On the other hand, the asymptotic terms corresponds to just the Coulomb-type gravitational scattering, which presumably does not probe the internal structure of the black holes. The hypergeometric nature of the near horizon equation remains for completely general black holes, with no extremality assumed. It is tempting to interpret this feature as a $\mathrm{SL}(2,\mathbb{R})$ symmetry as well, albeit one that is broken by coupling to the asymptotic space. This could signal the presence of a Virasoro algebra, even when there is no AdS-space at all. Seeing that the U(1) isometry of Kerr is enhanced to $\mathrm{SL}(2,\mathbb{R})$ and further to *Virasoro*, it is possible that the $\mathrm{U}(1)\times\mathrm{U}(1)$ isometry of general black holes might be enhanced to $\mathrm{SL}(2,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$ and on to *Virasoro*^{2}. One tangible piece of evidence for this structure is the remarkable quantization rule [17, 26]

$$\frac{1}{(8\pi G_4)^2}A_+A_- = \text{integer}\,, \tag{1.2}$$

satisfied by the outer/inner horizon area in an astonishing variety of examples. We will not pursue this wider perspective further in this paper but it is clearly one of the underlying motivations.

This paper is organized as follows. In section 2 we review the computation of greybody factors for 4D black holes with charges. We emphasize the verification of the matching condition for rotating black holes. In section 3 we turn to the greybody factors for 5D black holes, with charges and two independent angular momenta. In section 4 we discuss the CFT interpretation of these scattering amplitudes. Finally, in section 5, we situate Kerr/CFT relative to the CFTs describing more general black holes. In particular we relate the temperatures that appear naturally in the greybody factors to the Frolov-Thorne temperature employed in Kerr/CFT.

2 Greybody factors for 4D rotating black holes

In this section we review the Klein-Gordon equation in the background of the general 4D rotating black holes with charges. We discuss the matching procedure that leads to its solution, with emphasis on the extreme rotating limit.

2.1 The wave equation

We consider the general asymptotically flat 4D black hole with rotation, and also four independent U(1) charges. The solution was constructed in [27]. Following [18] we present

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the massless Klein-Gordon equation as

$$\left[4\frac{\partial}{\partial x}\left(x^2-\frac{1}{4}\right)\frac{\partial}{\partial x}+\frac{1}{x-\frac{1}{2}}\left(\frac{\omega}{\kappa_+}-m\frac{\Omega}{\kappa_+}\right)^2-\frac{1}{x+\frac{1}{2}}\left(\frac{\omega}{\kappa_-}-m\frac{\Omega}{\kappa_+}\right)^2+4\tilde{j}(\tilde{j}+1)\right.$$
$$\left.+8G_4xM\Delta\omega^2+x^2\Delta^2\omega^2\right]\Phi=0\ . \quad (2.1)$$

We employ the radial coordinate

$$x=\frac{r-\frac{1}{2}(r_++r_-)}{r_+-r_-}\,, \quad (2.2)$$

which is designed so that the two horizons

$$r_\pm=\frac{1}{4}(\mu\pm\sqrt{\mu^2-l^2})\,, \quad (2.3)$$

are at $x=\pm\frac{1}{2}$. The overall scale of the black hole is set by $r_++r_-=\frac{1}{2}\mu$. The departure from extremality is encoded in

$$\Delta=2(r_+-r_-)=\sqrt{\mu^2-l^2}\ . \quad (2.4)$$

We have assumed that the dependence of the wave function on the temporal and angular Killing vectors is

$$\Phi\propto e^{-i\omega t'+im\phi'}\,, \quad (2.5)$$

and we replaced the derivatives $\partial_{t'}$ and $\partial_{\phi'}$ in the Laplacian accordingly.

The dependence of the wave function on the polar coordinate θ is determined by the angular operator

$$\tilde{\Lambda}=-\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}+\frac{m^2}{\sin^2\theta}-\frac{1}{16}l^2\omega^2\cos^2\theta-\frac{1}{16}\mu^2\omega^2\left(1+\sum_{i<j}\cosh 2\delta_i\cosh 2\delta_j\right)\,, \quad (2.6)$$

For the purposes of the radial equation we can think of this operator as a constant (its eigenvalue):[1]

$$\tilde{\Lambda}\to\tilde{j}(\tilde{j}+1)\ . \quad (2.7)$$

In the special case of low energy $\mu\omega\ll\tilde{j}+\frac{1}{2}$ (which for near extreme Kerr implies also $l\omega\ll\tilde{j}+\frac{1}{2}$), the angular wave function is just a spherical harmonic with angular momentum $j=\tilde{j}$. We will in fact not assume low energy and so the generalized angular momentum $\tilde{j}$ is just a separation constant defined through (2.7). It takes on a sequence of discrete values that are not necessarily integral.[2]

[1] In [17] we used the notation $\zeta=\frac{1}{2}+\tilde{j}$. The recent work [10] similarly used $\beta=\frac{1}{2}+\tilde{j}$.

[2] Although the eigenvalue $\tilde{j}(\tilde{j}+1)$ must be real, there are parameters for which $\tilde{j}$ becomes complex. In [10] this possibility was interpreted as a genuine instability, interpreted in bulk as Schwinger pair production [28, 29]. Our computation applies only when $\tilde{j}$ is real.

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2.2 Parametric representation of black hole variables

We use a parametric form for the physical variables of the black holes

$$
\begin{aligned}
8G_4M &= \frac{1}{2}\mu\sum_{i=1}^{4}\cosh 2\delta_i\,, \\
8G_4Q_i &= \frac{1}{2}\mu\sinh 2\delta_i\,, \\
8G_4J &= \frac{1}{2}\mu l\left(\prod_{i=1}^{4}\cosh\delta_i - \prod_{i=1}^{4}\sinh\delta_i\right)\,.
\end{aligned}
\tag{2.8}
$$

The variables μ, l have dimension of length. Our charges Q_i have dimension of mass while the angular momentum J is dimensionless. The special case of Kerr-Newman black holes corresponds to having just one charge $Q \equiv \frac{1}{2}Q_i$ (for any i).

The surface accelerations $\kappa_\pm$ of the outer and inner horizons are encoded in the R- and L-temperatures $T_{R,L} = \beta_{R,L}^{-1}$ with the parametric form

$$
\begin{aligned}
\beta_R &= \frac{2\pi}{\kappa_+} + \frac{2\pi}{\kappa_-} = \frac{2\pi\mu^2}{\sqrt{\mu^2 - l^2}}\left(\prod_i\cosh\delta_i + \prod_i\sinh\delta_i\right)\,, \\
\beta_L &= \frac{2\pi}{\kappa_+} - \frac{2\pi}{\kappa_-} = 2\pi\mu\left(\prod_i\cosh\delta_i - \prod_i\sinh\delta_i\right)\,.
\end{aligned}
\tag{2.9}
$$

The (inverse) Hawking temperature are given terms of these as

$$
T_H^{-1} = \beta_H = \frac{2\pi}{\kappa_+} = \frac{1}{2}(\beta_R + \beta_L)\,. \tag{2.10}
$$

The angular velocity is parametrized as

$$
\frac{1}{\kappa_+}\Omega = \frac{l}{\sqrt{\mu^2 - l^2}}\,. \tag{2.11}
$$

For easy reference we also record two equivalent expressions for the black hole entropy

$$
\begin{aligned}
S &= 2\pi\left[\frac{1}{16G_4}\mu^2\left(\prod_i\cosh\delta_i + \prod\sinh\delta_i\right) + \sqrt{\frac{1}{256G_4^2}\mu^4\left(\prod_i\cosh\delta_i - \prod\sinh\delta_i\right)^2 - J^2}\right] \\
&= \frac{2\pi}{8G_4}\left[\frac{1}{2}\mu^2\left(\prod_i\cosh\delta_i + \prod\sinh\delta_i\right) + \frac{1}{2}\mu\sqrt{\mu^2 - l^2}\left(\prod_i\cosh\delta_i - \prod\sinh\delta_i\right)\right].
\end{aligned}
\tag{2.12}
$$

2.3 Solving the wave equation

We solve the radial equation (2.1) one region at a time, and then patch the partial solutions together for the complete wave function.

The near horizon region of the black hole involves all terms in (2.1) except those linear and quadratic in the radial coordinate x. The solution to this part of the equation is

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essentially a hypergeometric function[3]

$$\Phi_{\rm NH} = \left(\frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right)^{-i\frac{\beta_H}{4\pi}(\omega-m\Omega)} \left(x+\frac{1}{2}\right)^{-1-\tilde{j}} \times F\left(1+\tilde{j}-\frac{i}{2\pi}\left(\beta_R\frac{\omega}{2}-\beta_H m\Omega\right), 1+\tilde{j}-\frac{i\beta_L\omega}{4\pi}, 1-i\frac{\beta_H}{2\pi}(\omega-m\Omega), \frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right) . \quad (2.13)$$

The complex conjugate expression is a linearly independent solution.

The asymptotic region of the black hole involves just the terms that are constant or increase as a function of the radial coordinate x. The solution to this part of the equation alone is essentially Kummer's function, as usual for scattering on a potential with a long range force ($1/r$-component) and a centrifugal barrier ($1/r^2$-component).

In favorable cases there is a "matching" region where both the near horizon and the asymptotic approximation apply. In this region the radial equation involves just the generalized angular momentum barrier $\tilde{j}(\tilde{j}+1)$ and the kinetic energy. Accordingly, the near horizon wave function (2.13) and the asymptotic wave function both take the same, simple form

$$\Phi_{\rm matching} = a x^{\tilde{j}} \ . \quad (2.14)$$

The complementary solution $\sim x^{-1-\tilde{j}}$ is negligible. Therefore the coefficient a appearing in each of the two partial solutions of the wave equation must be the same, leading to the complete wave function.

In the case of absorption by the black hole, boundary conditions at the outer horizon are chosen such that there is no outgoing wave there. This determines the coefficient a in the matching region, and so the resulting fluxes in the asymptotic region. The result for the absorption cross-section found using the steps outlined above is [18][4]

$$\begin{aligned}\sigma_{\rm abs}(\omega) &= \frac{\pi(2\tilde{j}+1)}{\omega^2}\cdot\frac{2\beta_H(\omega-m\Omega)}{\pi\Delta\omega}\cdot\left|\frac{({\rm ampl})_0}{({\rm ampl})_\infty}\right|^2 \\ &= \frac{(\Delta\omega)^{1+2\tilde{j}}}{\omega^2}\sinh\frac{\beta_H(\omega-m\Omega)}{2}\left|\Gamma\left(1+\tilde{j}-\frac{i\beta_L\omega}{4\pi}\right)\Gamma\left(1+\tilde{j}-\frac{i}{2\pi}\left(\beta_R\frac{\omega}{2}-\beta_H m\Omega\right)\right)\right|^2 \\ &\quad\times\frac{(2\tilde{j}+1)}{\Gamma(2\tilde{j}+1)^2\Gamma(2\tilde{j}+2)^2}e^{2\pi G_4 M\omega}|\Gamma(1+\tilde{j}+2iG_4M\omega)|^2 \ . \end{aligned} \quad (2.15)$$

As we have emphasized, the only assumption we have made in finding this expression is the existence of a suitable matching region. Before turning to the analysis of this expression we therefore need to justify that assumption. That is what we turn to next.

[3]We have taken the azimuthal quantum number m into account by using the replacements $\beta_R\omega/2 \to \beta_R\omega/2-\beta_H m\Omega$, $\beta_H\omega \to \beta_H(\omega-m\Omega)$ noted in footnote 8 of [18].

[4]The absorption cross-section is not just the ratio of fluxes at the horizon and asymptotically: there is also the overlap with a plane wave. For the numerical factor we use (without detailed justification) the standard result also when $\tilde{j}$ is not an integer.

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2.4 Matching region in the near extreme Kerr limit

We are interested in the near extreme Kerr limit $\mu \sim l$ with the scale μ arbitrary:

$$\frac{\mu}{\Delta} \equiv \frac{\mu}{\sqrt{\mu^2 - l^2}} \gg 1 \ . \tag{2.16}$$

For the purposes of estimates we take charge parameters $\delta_i \sim 1$. The estimates are valid also for $\delta_i = 0$, but extreme limits that involve $\delta_i \gg 1$ along with (2.16) require additional considerations. In the near extreme limit the inverse temperatures $\beta_R, \beta_H \gg \mu$, but $\beta_L \sim \mu$ and $\Omega \sim \mu^{-1}$.

As discussed in the previous subsection, the matching region is "far away" from the near horizon perspective so we must require that the near horizon terms must be subleading there. Rewriting the near horizon expressions we find the condition

$$\frac{1}{x^2 - \frac{1}{4}} \left(\frac{\beta_H}{2\pi}(\omega - m\Omega) \right)^2 + \frac{1}{x + \frac{1}{2}} \left(\frac{\beta_L \omega}{2\pi} \right) \left(\frac{\beta_H}{\pi}(\omega - m\Omega) - \frac{\beta_L \omega}{2\pi} \right) \ll \tilde{j}(\tilde{j}+1) \, . \tag{2.17}$$

Thus for them to be subleading one requires:

$$\begin{aligned} \frac{1}{x^2} \cdot \beta_H^2 (\omega - m\Omega)^2 &\ll \tilde{j}(\tilde{j}+1) \, , \\ \frac{1}{x}(\beta_L \omega) \cdot |2\beta_H(\omega - m\Omega) - \beta_L \omega| &\ll \tilde{j}(\tilde{j}+1) \ . \end{aligned} \tag{2.18}$$

However, the matching region should nevertheless be "near the black hole" from the point of view of the asymptotically flat region. Thus the terms in (2.1) that are linear or quadratic in x should be negligible. This condition can be expressed as the inequalities

$$\begin{aligned} x^2 \cdot \frac{\Delta^2}{\mu^2} \mu^2 \omega^2 &\ll \tilde{j}(\tilde{j}+1) \, , \\ x \cdot \frac{\Delta}{\mu} \mu^2 \omega^2 &\ll \tilde{j}(\tilde{j}+1) \ . \end{aligned} \tag{2.19}$$

We need to establish that there is a range of $x \gg 1$ satisfying both (2.18) and (2.19).

For $x \gg 1$ a sufficient condition for (2.18) to be satisfied is:

$$\begin{aligned} \beta_H(\omega - m\Omega) &\sim 1 \, , \\ \beta_L \omega &\sim 1 \, . \end{aligned} \tag{2.20}$$

The first requirement requires that we probe the energies, which are natural for the co-rotating observer, and the second requirement further constrains the energy regime to be of order $\mathcal{O}(\mu^{-1})$ (recall $\beta_L = \mathcal{O}(\mu)$). Thus, we consider

$$\mu\omega \sim m \sim 1 \ . \tag{2.21}$$

Recalling that $\mu\Omega \sim 1$ in the extreme limit (2.16) we can do this with the difference $\omega - m\Omega$ tuned such that (2.20) is satisfied even though $\beta_H \gg \mu$.

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The energy and azimuthal quantum number of the scalar field contribute to the angular operator (2.6). With these scales taken as (2.21) it is natural to assume the generalized angular momentum $\tilde{j} \sim 1$ as well.

At this point we have specified the properties of the scalar field probe completely. It is then a simple matter to verify that the matching conditions (2.18)–(2.19) are solved in the range

$$1 \ll x \ll \frac{\mu}{\sqrt{\mu^2 - l^2}} \ . \tag{2.22}$$

2.5 A formal decoupling limit

It is instructive to revisit the limits we consider by comparing with a formal decoupling in real space, generalizing the NHEK limit [30] to the near extreme limit with charges.

Introducing

$$\epsilon\lambda = \frac{1}{2}(r_+ - r_-) = \frac{1}{4}\sqrt{\mu^2 - l^2}\,, \tag{2.23}$$

the near extreme limit is defined as $\lambda \to 0$ with the excitation scale ϵ kept fixed along with the horizon scale $\frac{1}{2}(r_+ + r_-) = \frac{1}{2}\mu$.

We focus on the near horizon region by introducing the scaling coordinate U through

$$r = \frac{1}{2}(r_+ + r_-) + \lambda U \ . \tag{2.24}$$

We keep U fixed as $\lambda \to 0$. The dimensionless coordinate $x = U/2\epsilon$ introduced in (2.2) also remains a good coordinate in the scalling limit.

In the wave equation (2.1) we assumed the form (2.5) for the wave function. In other words, we made the replacements

$$\begin{aligned} \frac{\partial}{\partial t'} &\to -i\omega\,, \\ \frac{\partial}{\partial \phi'} &\to im\ . \end{aligned} \tag{2.25}$$

Note that the asymptotic coordinates are defined with a prime from the outset. The near horizon observer more naturally employ the comoving coordinates

$$\begin{aligned} t &= \lambda t'\,, \\ \phi &= \phi' - \Omega t'\ . \end{aligned} \tag{2.26}$$

In these coordinates

$$\omega - m\Omega \to i(\partial_{t'} + \Omega\partial_{\phi'}) = i\lambda\partial_t \to \lambda\omega_{\rm com}\ . \tag{2.27}$$

The comoving energy $\omega_{\rm com}$ is kept finite in the scaling limit.

The surface accelerations $\kappa_\pm \sim \lambda$ so all the terms in the first line of the wave equation (2.1) remain finite in the scaling limit. In contrast, the asymptotic terms in the second line of (2.1) scale to zero. The scaling limit thus isolates the near horizon region, including the matching region, while the asymptotically flat region decouples as $\lambda \to 0$. It is therefore sensible to propose a theory that controls the near horizon region alone.

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2.6 Superradiance

At this point we have established the existence of a suitable matching region in the case of extremal Kerr (2.16). This means the absorption cross-section (2.15) applies in this case.

The non-trivial frequency dependence in the absorption cross-scetion (2.15) all remains in the limit discussed in the previous two subsections: there is no further simplification beyond the one due to the existence of a matching region.

In the formal decoupling limit $\lambda \to 0$ the absorption cross-section in fact vanishes, because the overall prefactor scales to zero. Thus the black hole does not absorb incoming waves, nor does it emit particles. This limit is therefore truly a decoupling limit.

It is interesting and surprising that the absorption cross-section (2.15) may be negative

$$\sigma_{\rm ab}(\omega < m\Omega) < 0 \ . \tag{2.28}$$

In this situation the amplitude of the wave reflecting from the black hole is larger than the incoming wave. This phenomenon is known as super-radiance [31–33].

It is interesting to trace the origin of super-radiance in our set-up. Very near the (outer) horizon at $x \sim \frac{1}{2}$ the radial wave function (2.13) reduces to

$$\Phi_{\rm NH}\left(x - \frac{1}{2} \ll 1\right) \sim \left(x - \frac{1}{2}\right)^{-i\frac{\beta_H}{4\pi}(\omega - m\Omega)} \ . \tag{2.29}$$

An exponent with negative imaginary part corresponds to an incoming wave, as one expects for absorption by the black hole. However, for $\omega < m\Omega$ the exponent has positive imaginary part. Then the flux is flowing out from the horizon so that, at infinity, more flux is reflected than is send in.

2.7 The emission spectrum

The spectrum of emitted Hawking radiation follows from the absorption cross-section by detailed balance. It becomes

$$\begin{aligned}
\Gamma_{\rm em}(\omega) &= \sigma_{\rm abs}(\omega)\frac{1}{e^{\beta_H(\omega - m\Omega)-1}}\frac{d^3k}{(2\pi)^3} \\
&= \frac{(\Delta\omega)^{1+2\tilde{j}}}{2\omega^2}e^{\beta_H(\omega-m\Omega)/2}\left|\Gamma\left(1+\tilde{j}-\frac{i\beta_L\omega}{4\pi}\right)\Gamma\left(1+\tilde{j}-\frac{i}{2\pi}\left(\beta_R\frac{\omega}{2}-\beta_H m\Omega\right)\right)\right|^2 \\
&\quad \times \frac{(2\tilde{j}+1)}{\Gamma(2\tilde{j}+1)^2\Gamma(2\tilde{j}+2)^2}e^{2\pi G_4 M\omega}|\Gamma(1+\tilde{j}+2iG_4M\omega)|^2\frac{d^3k}{(2\pi)^3} \ .
\end{aligned} \tag{2.30}$$

The emission spectrum does not exhibit superradiance: superradiance is stimulated emission so it relies on the incoming quanta as well. However, emission is more convenient for the discussion of the CFT description.

The frequency dependence in the final line of (2.30) is due to the long range nature of the interaction. This term is present for all processes involving $1/r$ forces, including atomic and nuclear scattering. Although it arises from a hypergeometric function it can presumably not be interpreted as due to an underlying CFT. In section 4 we will therefore seek to understand just the frequency dependence in the first line of (2.30).

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3 Near-extreme Kerr black holes in D=5

In this section we carry out the analysis of near extreme Kerr black holes in five dimensions. We maintain all three U(1) charges and two independent angular momenta.

3.1 The scalar wave equation

The asymptotically flat black hole solution in 5D with independent values for the two angular momenta and also three independent charges was found in [34]. The corresponding Klein-Gordon equation was presented in [17] as

$$\left[4\frac{\partial}{\partial x}\left(x^2-\frac{1}{4}\right)\frac{\partial}{\partial x}+\frac{1}{x-\frac{1}{2}}\left(\frac{\omega}{\kappa_+}-m_R\frac{\Omega_R}{\kappa_+}-m_L\frac{\Omega_L}{\kappa_+}\right)^2-\frac{1}{x+\frac{1}{2}}\left(\frac{\omega}{\kappa_-}-m_R\frac{\Omega_R}{\kappa_+}+m_L\frac{\Omega_L}{\kappa_+}\right)^2 -\tilde{j}(\tilde{j}+2)+x\Delta\omega^2\right]\Phi_0=0\ . \tag{3.1}$$

The radial coordinate

$$x=\frac{r^2-\frac{1}{2}(r_+^2+r_-^2)}{r_+^2-r_-^2}\ , \tag{3.2}$$

is designed to put the horizons

$$r_\pm^2=\frac{1}{2}\left(\mu\pm\sqrt{(\mu-(l_1-l_2)^2)(\mu-(l_1+l_2)^2)}\right)\ , \tag{3.3}$$

at $x=\pm\frac{1}{2}$ for all values of the black hole parameters. The departure from extremality (which may be arbitrary at this point) is encoded in

$$\Delta=r_+^2-r_-^2=\sqrt{(\mu-(l_1-l_2)^2)(\mu-(l_1+l_2)^2)}\ . \tag{3.4}$$

The full angular Laplacian for the problem is

$$\hat{\Lambda}=-\frac{1}{\sin 2\theta}\frac{\partial}{\partial\theta}\sin 2\theta\frac{\partial}{\partial\theta}-\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}-\frac{1}{\cos^2\theta}\frac{\partial^2}{\partial\psi^2}+(l_1^2+l_2^2)\omega^2+(l_2^2-l_1^2)\omega^2\cos 2\theta-M\omega^2\ . \tag{3.5}$$

We denote the eigenvalue of this operator $\tilde{j}(\tilde{j}+2)$. Accordingly, we inserted this value of the angular momentum barrier in the radial equation (3.1). At low energy $l_{1,2}\omega^2, M\omega^2\ll 1$ our notation $\tilde{j}$ reduces to the usual angular momentum j, which in that limit labels the quadratic Casimirs of the rotation group SO(4) $\simeq$ SU(2) $\times$ $SU(2)$. However, we will not assume that the energy is small and so the generalized angular momentum $\tilde{j}$ is just a notation for the separation constant of the Klein-Gordon equation.[5]

[5] As in 4D, $\tilde{j}(\tilde{j}+2)$ must be real, but in general this combination can be less than -1 so that $\tilde{j}$ may acquire an imaginary part.

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3.2 Parametric form of black hole variables

In the general case with three U(1) charges it is essential that we employ the parametric representation of black hole variables [34]

$$\begin{aligned}
\frac{4G_5}{\pi}M &= \frac{1}{2}\mu\sum_{i=1}^{3}\cosh 2\delta_i\,,\\
\frac{4G_5}{\pi}Q_i &= \frac{1}{2}\mu\sinh 2\delta_i\,, \quad (i=1,2,3)\,,\\
\frac{4G_5}{\pi}J_{R,L} &= \frac{1}{2}\mu(l_1\pm l_2)\left(\prod_{i=1}^{3}\cosh\delta_i\mp\prod_{i=1}^{3}\sinh\delta_i\right)\,.
\end{aligned} \tag{3.6}$$

Note that in 5D the scale μ has dimension of length *squared*. The parametric angular momenta $l_{1,2}$ are lengths and the parametric charges are δ_i dimensionless.[6]

The surface accelerations $\kappa_\pm$ in the radial equation (3.1) are equivalent to the inverse temperatures

$$\beta_{R,L} = \frac{2\pi}{\kappa_+}\pm\frac{2\pi}{\kappa_-}\,, \tag{3.7}$$

which in turn have the parametric form

$$\begin{aligned}
\beta_L &= \frac{2\pi\mu}{\sqrt{\mu-(l_1-l_2)^2}}\left(\prod_i\cosh\delta_i-\prod_i\sinh\delta_i\right)\,,\\
\beta_R &= \frac{2\pi\mu}{\sqrt{\mu-(l_1+l_2)^2}}\left(\prod_i\cosh\delta_i+\prod_i\sinh\delta_i\right)\,.
\end{aligned} \tag{3.8}$$

The inverse Hawking temperature is $\beta_H=\frac{2\pi}{\kappa_+}$.

The angular velocities in the radial equation (3.1) have the parametric forms

$$\begin{aligned}
\beta_H\Omega_L &= \frac{2\pi(l_1-l_2)}{\sqrt{\mu-(l_1-l_2)^2}}\,,\\
\beta_H\Omega_R &= \frac{2\pi(l_1+l_2)}{\sqrt{\mu-(l_1+l_2)^2}}\,.
\end{aligned} \tag{3.9}$$

For later reference we also record the black hole entropy

$$S = 2\pi\sqrt{\frac{\pi^2}{64G_5^2}\mu^3\left(\prod_i\cosh\delta_i+\prod\sinh\delta_i\right)^2-J_L^2}+2\pi\sqrt{\frac{\pi^2}{64G_5^2}\mu^3\left(\prod_i\cosh\delta_i-\prod\sinh\delta_i\right)^2-J_R^2}\,. \tag{3.10}$$

[6] We indicate Newton's constant explicitly. For the value $G_5=\frac{\pi}{4}$ the formulae simplify and Q_i becomes integral in the simplest string theory embedding (see eg. [35]).

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3.3 Wave functions and greybody factors

The radial equation (3.1) cannot be solved analytically in general. However, in the near horizon region where the term linear in x can be neglected the equation is hypergeometric with solution[17][7]

$$\Phi_{\rm NH}(x) = \left(\frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right)^{-i\frac{\beta_H}{4\pi}(\omega-m_L\Omega_L-m_R\Omega_R)}\left(x+\frac{1}{2}\right)^{-1-\frac{1}{2}\tilde{j}}\times F\left(1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}\left(\frac{\beta_R\omega}{2}-\beta_H m_R\Omega_R\right),\right.$$
$$\left.1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}\left(\frac{\beta_L\omega}{2}-\beta_H m_L\Omega_L\right), 1-i\frac{\beta_H}{2\pi}(\omega-m_L\Omega_L-m_R\Omega_R), \frac{x-\frac{1}{2}}{x+\frac{1}{2}}\right). \quad (3.11)$$

The wave function was chosen with incoming boundary conditions. The complex conjugate wave function is a linearly independent solution, with outgoing boundary condition. The asymptotic behavior of (3.11) for large x takes the form

$$\Phi_{\rm NH}(x) \sim a x^{\frac{1}{2}\tilde{j}} \ . \quad (3.12)$$

The solution in the asymptotic region where the horizon terms with singularities as $x = \pm\frac{1}{2}$ can be neglected is also simple: it is just a Bessel function. In the short distance limit this asymptotic wave function takes the same form as (3.12). In cases where an overlapping regime of applicability of the two regimes can be established the coefficient a for the two regional wave functions must agree, and then the full wave function follows. Comparing the asymptotic flux to the one at the horizon, we find the transmission coefficient

$$|T_{\tilde{j}}|^2 = \beta_H(\omega-m_L\Omega_L-m_R\Omega_R)\left(\frac{\sqrt{\Delta}\omega}{2}\right)^{2+2\tilde{j}}$$
$$\times\left|\frac{\Gamma(1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}(\beta_L\frac{\omega}{2}-\beta_H m_L\Omega_L))\Gamma(1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}(\beta_R\frac{\omega}{2}-\beta_H m_R\Omega_R))}{\Gamma(\tilde{j})\Gamma(1+\tilde{j})\Gamma(1-\frac{i}{2\pi}\beta_H(\omega-m_L\Omega_L-m_R\Omega_R))}\right|^2 \quad (3.13)$$

Expanding one of the Γ-functions, the absorption cross-section becomes[8]

$$\sigma_{\rm abs}(\omega) = \frac{8\pi}{\omega^3}\sinh\left(\frac{1}{2}\beta_H(\omega-m_L\Omega_L-m_R\Omega_R)\right)\left(\frac{\sqrt{\Delta}\omega}{2}\right)^{2+2\tilde{j}}\frac{(\tilde{j}+1)^2}{|\Gamma(\tilde{j})\Gamma(1+\tilde{j})|^2}$$
$$\times\left|\Gamma\left(1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}\left(\beta_L\frac{\omega}{2}-\beta_H m_L\Omega_L\right)\right)\Gamma\left(1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}\left(\beta_R\frac{\omega}{2}-\beta_H m_R\Omega_R\right)\right)\right|^2 . \quad (3.14)$$

[7] The notation of [17] is $\xi = 1+\frac{1}{2}\tilde{j}$.

[8] The transmission coefficient and the cross-section are related by the overlap between our wave function in spherical coordinates and a plane wave. This is difficult to compute because the solutions to the angular equation (3.5) are involved when there is no spherical symmetry. Guided by spherical symmetry, we use the overlap $4\pi(\tilde{j}+1)^2/\omega^3$, knowing that this expression should receive small corrections.

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3.4 The near extreme limit and matching conditions

The 5D near extreme Kerr limit takes one of the two angular momenta large, keeping the other at moderate values. Without loss of generality, we take $J_R \sim J_{R,\max}$, with J_L arbitrary. In our parametric notation we take

$$\frac{\sqrt{\mu}}{\sqrt{\mu-(l_1+l_2)^2}} \gg 1 \ . \tag{3.15}$$

The variables are otherwise not constrained so we can estimate $\mu-(l_1-l_2)^2 \sim \mu$ for the combination that controls the other angular momentum (J_L). The non-extremality parameter (3.4) becomes

$$\Delta \sim 2\sqrt{l_1 l_2}\sqrt{\mu-(l_1+l_2)^2} \leq \sqrt{\mu}\sqrt{\mu-(l_1+l_2)^2} \ll \mu \tag{3.16}$$

in the limit (3.15). In our limit $\beta_H \sim \beta_R \gg \beta_L$. Also, $\Omega_R \sim \mu^{-1/2}$ and $\Omega_L \sim \beta_H^{-1}$ so $\Omega_R \gg \Omega_L$. As in 4D we take the charge parameters $\delta_i \sim 1$ in our estimates.

The matching region is a range of x where the angular momentum barrier dominates the near horizon terms. Rewriting the near horizon expressions we find the condition:

$$\begin{aligned}&\frac{1}{x^2-\frac{1}{4}}\left(\frac{\beta_H}{2\pi}(\omega-m_R\Omega_R-m_L\Omega_L)\right)^2 + \\ &\frac{1}{x+\frac{1}{2}}\left(\frac{\beta_L\omega}{2\pi}-\frac{\beta_H m_L\Omega_L}{\pi}\right)\left(\frac{\beta_H}{\pi}(\omega-m_R\Omega_R-m_L\Omega_L)-\left(\frac{\beta_L\omega}{2\pi}-\frac{\beta_H m_L\Omega_L}{\pi}\right)\right) \ll \tilde{j}(\tilde{j}+2)\,.\end{aligned} \tag{3.17}$$

Since $\Omega_L \ll \Omega_R$ and $\tilde{j} \sim 1$, sufficient conditions for these terms to be subleading for $x \gg 1$ are:

$$\beta_H(\omega-m_R\Omega_R) \sim 1\,, \quad \beta_L\omega-\beta_H m_L\Omega_L \sim 1 \ . \tag{3.18}$$

The first condition is the most delicate since β_H is large. We satisfy it by focussing on modes with their natural energy and azimuthal quantum number, but a cancellation so that (3.18) is satisfied. The second condition is almost automatic since neither β_L or $\beta_H\Omega^L$ is large in the near extreme limit. In formulae, we take:

$$\sqrt{\mu}\omega \sim m_R \sim m_L \sim 1\,, \tag{3.19}$$

with the precise values of ω, m_R tuned so that (3.18) remains satisfied even though $\beta_H \gg \sqrt{\mu}$ in the limit (3.15).

In the matching region the angular momentum barrier must also dominate the term encoding asymptotic Minkowski space, ie. the term in (3.1) that is linear in x. This gives the condition

$$x\frac{\Delta}{\mu}(\mu\omega^2) \ll \tilde{j}(\tilde{j}+2) \ . \tag{3.20}$$

The natural magnitude for the generalized angular momentum is similarly $\tilde{j} \sim 1$, since the expression (3.5) receives contributions from terms of the order (3.19).

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We can now verify that the conditions (3.18)–(3.19) on the scalar wave are sufficient to satisfy the matching conditions (3.17) and (3.20) in the range

$$1 \ll x \ll \sqrt{\frac{\mu}{\mu - (l_1 + l_2)^2}} \ . \tag{3.21}$$

This is what is needed to justify the greybody factor (3.14).

As in 4D (section 2.5) we could formalize the estimates in this section such that the validity of the approximations are recast as a formal limit, rendering the near horizon region (including the matching region) properly decoupled from the asymptotically flat space. We will mostly refer to the approximate notation detailed in this section.

3.5 Superradiant greybody factors

At this point we have justified the use of the matching procedure for the 5D extreme black holes with charge. All the structure in the absorption cross-section (3.14) persists in the scaling limit, there are no further simplifications.

As in 4D, the absorption cross-section may turn negative, corresponding to superradiance. The condition for this phenomenon is

$$\beta_H(\omega - m_L\Omega_L - m_R\Omega_R) = \left(\beta_L\frac{\omega}{2} - \beta_H m_L\Omega_L\right) + \left(\beta_R\frac{\omega}{2} - \beta_H m_R\Omega_R\right) < 0 \ . \tag{3.22}$$

The two parenthesis in the last expression are both of order 1 in our scaling limit. Superradiance can therefore be realized in non-trivial ways in 5D: it can be due to level inversion in *either* the L *or* the R side.

3.6 The emission spectrum

We may recast the absorption cross-section (3.14) as an emission amplitude for Hawking radiation, using detailed balance. The result is

$$\begin{aligned}\Gamma_{\rm em}(\omega) &= \sigma_{\rm abs}(\omega)\frac{1}{e^{\beta_H(\omega - m_L\Omega_L - m_R\Omega_R)} - 1}\frac{d^4k}{(2\pi)^4} \\ &= \frac{4\pi}{\omega^3}\left(\frac{\sqrt{\Delta}\omega}{2}\right)^{2+2\tilde{j}}\frac{(\tilde{j}+1)^2}{|\Gamma(\tilde{j})\Gamma(1+\tilde{j})|^2}e^{-\frac{1}{2}\beta_H(\omega - m_L\Omega_L - m_R\Omega_R)} \\ &\quad\times\left|\Gamma\left(1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}\left(\beta_L\frac{\omega}{2}-\beta_H m_L\Omega_L\right)\right)\Gamma\left(1+\frac{1}{2}\tilde{j}-\frac{i}{2\pi}\left(\beta_R\frac{\omega}{2}-\beta_H m_R\Omega_R\right)\right)\right|^2\frac{d^4k}{(2\pi)^4} \ .\end{aligned} \tag{3.23}$$

In the case where U(1) charges and two independent angular momenta are included the four potentials $\beta_{R,L} and \beta_H\Omega_{R,L}$ are independent. This gives significant structure to the amplitude (3.23).

We have maintained the notation appropriate for the asymptotic observer. However, in the near horizon theory it is more natural to introduce the rescaled potential $\tilde{\beta_H} = \lambda\beta_H$ and the corresponding comoving energy $\omega_{\rm com} = \lambda(\omega - m_R\Omega_R)$ with the scaling parameter $\lambda \sim \frac{\sqrt{\mu^2-(l_1+l_2)^2}}{\mu}$ taken to be small. The rescaled quantities are finite even in the formal scaling limit $\lambda \to 0$.

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4 The CFT model

In this section we model the emission amplitudes from a microscopic point of view. The presentation follows our previous papers [17, 18, 36], now adapted to the Kerr/CFT context. In comparison with the recent work [10] we include all the overall frequency dependent factors. We also keep all four U(1) charges in the 4D theory, and we include both angular momenta in the 5D theory. These additional black hole parameters makes the general structure more transparent and makes the relation to the BPS cases clearer. We first consider the 5D theory, and then briefly the 4D case.

4.1 5D emission spectrum from CFT

The working assumption of the microscopic model is that the entire near horizon region, including the matching region, can be described by a dual CFT, generalizing the 4D Kerr/CFT [8].

That the near horizon region should be dual to some quantum field theory is suggested by the decoupling of this region from the asymptotically flat space. That the theory should be a CFT is made possible by the classical fields reducing to hypergeometric functions, which are the characters of the $\mathrm{SL}(2,\mathbb{R})$ group. The geometrical origin of the $\mathrm{SL}(2,\mathbb{R})$ is the isometry group the AdS_2 factor in the geometry, and the wave equation is the $\mathrm{SL}(2,\mathbb{R})$ Casimir.

In the description where the near horizon region is replaced by a CFT, the emission of quanta embodied in (3.23) is due to couplings

$$\Phi_{\mathrm{bulk}}\mathcal{O}^{(h,\bar{h})} \tag{4.1}$$

between bulk modes Φ_{bulk} and operators $\mathcal{O}^{(h,\bar{h})}$ in the CFT. The structure of the resulting emission depends primarily on the conformal weights $(h,\bar{h})$ of the operator. The value of the conformal weight

$$h=\bar{h}=1+\frac{1}{2}\tilde{j}\,, \tag{4.2}$$

can be read off from the asymptotic behavior (3.12) near the boundary of the near horizon region.

In situations where the black hole background has spherical symmetry the difference $h-\bar{h}$ measures the bulk spin s and so it is obvious that $h=\bar{h}$ for scalar fields in bulk (see e.g. [16, 37]). The Kerr black hole is not spherically symmetric and so $h=\bar{h}$ is not clear *a priori* [10].

The canonical thermal two-point function of chiral operator with conformal weight h is specified by the singularity $\sim z^{-2h}$ and the periodicty $2\pi\beta^{-1}$:

$$G^h_\beta(z)=\left(\frac{\pi/\beta}{\sinh\left(\pi z/\beta\right)}\right)^{2h} \tag{4.3}$$

The Fourier transform is

$$G^h_\beta\left(\frac{\omega}{2}\right)=\left(\frac{2\pi}{\beta}\right)^{2h-1}e^{-\beta\omega/4}\frac{1}{\Gamma(2h)}\left|\Gamma\left(h+\frac{i\beta\omega}{4\pi}\right)\right|^2 \tag{4.4}$$

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The two point function of an operator $\mathcal{O}^{(h,\bar{h})}$ with conformal weights (4.2) thus gives the contributions to the emission amplitude from the CFT operators:

$$\Gamma_{\rm em}(\omega) \propto \left(\frac{4\pi^2}{\beta_R\beta_L}\right)^{\tilde{j}+1} e^{-\beta_L(\omega-m_L\Omega_L)/4-\beta_R(\omega-m_R\Omega_R)/4} \times \left|\Gamma\left(1+\frac{1}{2}\tilde{j}+\frac{i}{2\pi}\left(\beta_R\frac{\omega}{2}-\beta_H m_R\Omega_R\right)\right)\right|^2 \cdot \left|\Gamma\left(1+\frac{1}{2}\tilde{j}+\frac{i}{2\pi}\left(\beta_L\frac{\omega}{2}-\beta_H m_L\Omega_L\right)\right)\right|^2 \tag{4.5}$$

This goes a long way towards accounting for the supergravity expression (3.23). As explained after (3.23), we maintain the notation appropriate for the asymptotic observer. Since the CFT knows only about comoving energies and rescaled temperatures, it is those combinations that appear in (4.5).

The details of the emission will depend on the coupling (4.1) between the CFT operator and the bulk field. If (4.1) is literally the coupling, the only additional frequency dependence is ω^{-1} from the standard normalization of the outgoing wave function $\Phi_{\rm bulk}$. However, generally the coupling must also include derivatives and numerical group theory factors (such as Γ-matrices) in order to ensure Lorentz invariance and other symmetries. At low energy, the coupling to spin j involves precisely j derivatives that act on the outgoing wave function, giving a factor ω^j in the amplitude, and the square of that in the probability [16, 38–40]. In Kerr/CFT there is not enough known about the dual theory that we can construct the coupling to bulk fields in any detail. Nevertheless, it is reasonable to expect such couplings to lead to an overall frequency dependence

$$\omega^{2\tilde{j}-1} \,. \tag{4.6}$$

It is the "far away" frequency ω rather than either of the "near horizon" (comoving) frequencies $\omega - m_{L,R}\Omega_{L,R}$ that enter in this factor. The reason is that the (generalized) derivatives in the coupling can be taken to act on the bulk wave function $\Phi_{\rm bulk}$ which only reaches into the matching region. Thus the coupling is sensitive to the deformation of the sphere due to rotation ($\tilde{j}$ rather than j) but not to the motion of the near horizon region.

The emission rate (4.5) in the microscopic model, with the prefactor (4.6), should be compared with the supergravity result (3.23). A useful relation is

$$\frac{\beta_R\beta_L\Delta}{2\pi} = \frac{4G_5}{\pi}\mathcal{L}_5 \,, \tag{4.7}$$

where we have introduced the length scale $\mathcal{L}_5$ through [17, 41]

$$\frac{4G_5}{\pi}\mathcal{L}_5 = 2\pi\mu^2\left(\prod_{i=1}^{3}\cosh^2\delta_i - \prod_{i=1}^{3}\sinh^2\delta_i\right) \,. \tag{4.8}$$

The two expressions depend identically on all black hole parameters. One formal discrepancy arises because the CFT expressions like (4.4) were written by convention in units where the CFT is defined on a space of unit length. The comparison determines that length scale as $\mathcal{L}_5$ given in (4.8).

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We do not have a derivation of this length scale from first principle in the present context. However, in the near BPS limit $\delta_{1,2} \gg 1$ the length scale depends on just two of the three charges, and on the length scale associated with the third charge. In the standard $D1 - D5 - KK$ duality frame, the scale becomes

$$\mathcal{L}_{5,\mathrm{BPS}} = 2\pi n_1 n_5 R \ . \tag{4.9}$$

This is the "long string scale", corresponding to maximal winding around the compact KK-circle [42]. It is this scale that controls emission amplitude in many simpler contexts (see eg [13, 38, 43, 44]). In the next section we discuss the corresponding length scale for near extreme Kerr.

We have not attempted to reproduce the overall numerical factors in the emission amplitude from a microscopic point of view. In the simplest case of low energy emission from a spherical symmetric black hole the numerical factor was understood long time ago [43]. There is also (at least) a partial understanding of the numerical factors pertaining to higher partial waves [16, 38, 39] but those depend on the explicit coupling between bulk modes and the CFT which is not available here.

The overall scaling of the amplitudes represents an interesting point. In the CFT amplitude (4.5) the overall normalization include $\beta^{-\tilde{j}-1}$. It is the CFT temperature that enters, so it would be more correct to write $\tilde{\beta}_R^{-\tilde{j}-1} = \lambda^{-\tilde{j}-1}\beta^{-\tilde{j}-1}$, in the notation after (3.23). In other words, the supergravity amplitude is suppressed in the scaling parameter λ, as one expects when the near horizon theory is fully decoupled; but the CFT amplitude is not suppressed, because everything is written in terms of rescaled variables.

4.2 4D emission spectrum from CFT

The microscopic model that gives an interpretation of the 4D supergravity emission amplitude is very similar to the 5D model so we shall just summarize the main formulae.

The conformal weight of the CFT operator that is responsible for the emission can be read off from the wave function (2.14) in the matching region [18]

$$h = \bar{h} = 1 + \tilde{j} \ . \tag{4.10}$$

The two point correlations function of the operator is again (4.4) in Fourier space. The normalization of the outgoing bulk wave function and the frequency dependence from the couplings combine to give an overall frequency dependence

$$\omega^{2\tilde{j}-1} \ . \tag{4.11}$$

Collecting these factors give the emission amplitude

$$\begin{aligned}\Gamma_{\mathrm{em}}(\omega) \propto & \left(\frac{4\pi^2}{\beta_R\beta_L}\right)^{2\tilde{j}+1} \omega^{2\tilde{j}-1} e^{-\beta_H(\omega - m\Omega)/2} \\ & \times \left|\Gamma\left(1+\tilde{j}+\frac{i}{2\pi}\left(\beta_R\frac{\omega}{2} - \beta_H m\Omega\right)\right)\right|^2 \cdot \left|\Gamma\left(1+\tilde{j}+\frac{i}{4\pi}\beta_L\omega\right)\right|^2 \ .\end{aligned} \tag{4.12}$$

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The complete dependence on the frequency ω and the azimuthal quantum number m agrees precisely with (2.30) (except for the Coulomb factors in the last line (2.30) which should be neglected, as explained just after (2.30)).

The dependence on the black hole parameters can be compared by using the relation

$$\frac{\beta_R \beta_L \Delta}{2\pi} = 8G_4 \mathcal{L}_4 \,, \tag{4.13}$$

where we have introduced the length scale $\mathcal{L}_4$ through [17, 41]

$$8G_4 \mathcal{L}_4 = 2\pi\mu^3 \left(\prod_{i=1}^{4} \cosh^2 \delta_i - \prod_{i=1}^{4} \sinh^2 \delta_i \right) \,. \tag{4.14}$$

The dependence on the black hole parameters also agrees except that, as in 5D, we have normalized our CFT correlation functions so that they depend on just the physical temperatures, but the size of the spatial circle has been scaled out. The comparison determines that length scale as (4.14). As in 5D we interpret this scale as the "long string" scale [14, 45]. In the limit where the black hole is nearly BPS there are three large charges $\delta_{1,2,3} \gg 1$ and (4.14) reduces to

$$\mathcal{L}_{4,\mathrm{BPS}} = 2\pi n_1 n_2 n_3 R \,, \tag{4.15}$$

where R is the size of a physical compactification circle. The general expression (4.14) for the "long string scale" should be useful also away from the BPS limit. In the next section we make this expectation explicit in Kerr/CFT.

5 Features of the microscopic theory

In this section we extract some of the features of the microscopic theory. We focus on the 4D theory for easy comparison with other works, and just summarize the 5D formulae. The important point as that we include all charges to appreciate the full structure.

5.1 Phenomenological model for general 4D black holes

We start out very ambitiously, by writing the beginnings of a model for the *entire* class of 4D black holes we consider, including black holes that are nowhere near extremality. The working hypothesis is that all these black holes can be interpreted as a 2D CFT in a periodic box with some unknown radius $\mathcal{R}_4$, and that the entropy is captured by the standard high temperature expression:

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R)\mathcal{R}_4 \,. \tag{5.1}$$

The two temperatures $T_{L,R}$ we identify with the temperatures (2.9) that appear in the greybody factors, and the entropy of the left and right movers independently we take from the two terms in (2.12). These assumptions give expressions for the central charge in units of the box radius

$$c_L \mathcal{R}_4 = c_R \mathcal{R}_4 = 12 \cdot \frac{\mu^3}{16G_4} \left(\prod_{i=1}^{4} \cosh^2 \delta_i - \prod_{i=1}^{4} \sinh^2 \delta_i \right) \,. \tag{5.2}$$

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It is interesting that the central charge found this way is the same for the two chiralities.

Let us now specialize to black holes that are extreme due to their rotation. In this limit the entropy is exclusively due to the L-sector. It has been argued [8] that in this situation the temperature of the L-sector is the Frolov-Thorne temperature [46], which for general charges can be computed as

$$\beta_{\rm FT} = \frac{\partial}{\partial J} S(M = M_{\rm ext}) = \frac{2\pi J}{\sqrt{J^2 + \frac{1}{64G_4^2}\prod_{i=1}^4 Q_i}} \ . \tag{5.3}$$

This value is somewhat puzzling because it differs from the temperature β_L in (2.9) which, as we have seen, appears quite prominently in the physical greybody factors, even in the extreme limit. To resolve this tension we note that the Frolov-Thorne temperature defined in (5.3) is dimensionless. However, *the natural unit is the box radius*, so we can in fact identify the two proposed temperatures, after all:

$$\beta_{\rm FT} = \beta_L / \mathcal{R}_4 \ . \tag{5.4}$$

Moreover, this identification determines the box size as

$$\mathcal{R}_4 = \mu \left(\prod_{i=1}^4 \cosh \delta_i + \prod_{i=1}^4 \sinh \delta_i \right) \ . \tag{5.5}$$

The CFT is of course scale invariant so $\mathcal{R}_4$ has no meaning in the microscopic theory: only the complex structure encoded in β_{FT} makes sense. However, the identification of observables at infinity involves $\mathcal{R}_4$. Additionally, $\mathcal{R}_4 \sim \tilde{\beta}_R$, the rescaled temperature that does make sense in the CFT.

At this point the central charge determined from (5.2) becomes

$$c_L = c_R = 12 \cdot \frac{\mu^2}{16G_4} \left(\prod_{i=1}^4 \cosh \delta_i - \prod_{i=1}^4 \sinh \delta_i \right) = 12J \ . \tag{5.6}$$

The final equality followed from the relation between charges, mass and angular momentum in the extreme limit. The result for the central charge agrees with the well known one from Kerr/CFT. In particular it does not depend on the value of the U(1) charges. This suggests that all the U(1) charges are present in the CFT from the outset.

We are now ready to reconsider the length scale $\mathcal{L}_4$ that was extracted from the greybody computations. Combining the formulae above, we find

$$\mathcal{L}_4 = J \cdot 2\pi \mathcal{R}_4 \ . \tag{5.7}$$

The effective length that appears in the scattering is therefore essentially the same as the box size inferred from the simplest thermodynamic model. The only difference is a rescaling related to the background angular momentum. This rescaling is reminiscent of the "long string" rescaling (4.15) of BPS black holes. Our result is a quantitative prediction for a similar phenomenon in Kerr/CFT.

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5.2 The 5D model

In D=5 the entropy formula (3.10) can also quite generally be cast in the form (5.1) with two temperatures $T_{L,R}$ identified with those appearing in the greybody factors (3.8). This procedure gives the central charges in units of the box radius $\mathcal{R}_5$ are

$$c_L\mathcal{R}_5 = c_R\mathcal{R}_5 = 12\cdot\frac{\pi\mu^2}{8G_5}\left(\prod_{i=1}^{3}\cosh^2\delta_i - \prod_{i=1}^{3}\sinh^2\delta_i\right) . \tag{5.8}$$

Again, the central charges are the same for both chiralities.

When specializing to the extreme black holes, the entropy has only a contribution for the L-sector. The Frolov-Thorne temperature along the dominant (R) motion becomes:

$$\beta_{\rm FT} = \frac{\partial}{\partial J_R}S(M=M_{\rm ext}) = \frac{2\pi J_R}{\sqrt{J_R^2 - J_L^2 + \frac{4G_5}{\pi}\prod_{i=1}^{3}Q_i}} . \tag{5.9}$$

We can identify the greybody temperature β_L with the Frolov-Thorne temperature (in units of a box size) β_{FT} by introducing the box-size

$$\mathcal{R}_5 = \frac{\beta_L}{\beta_{\rm FT}} = 2\pi\mu^{1/2}\left(\prod_{i=1}^{3}\cosh\delta_i + \prod_{i=1}^{3}\sinh\delta_i\right) , \tag{5.10}$$

As in 5D, the box size if of order the "small" temperature, in units if the scaling variable $\mathcal{R}_5 \sim \tilde{\beta}_R$.

The central charges determined from (5.8) now become:

$$c_L = c_R = 12\cdot\frac{\mu^{3/2}}{16G_5}\left(\prod_{i=1}^{3}\cosh\delta_i - \prod_{i=1}^{3}\sinh\delta_i\right) = 12J_R \tag{5.11}$$

as expected. These expressions for the central charges are compatible with those found in [24] where Kerr/CFT techniques were employed.[9]

Finally, we can now derive the length scale $\mathcal{L}_5$ that was extracted from the greybody computations. Combining the formulae above, we find

$$\mathcal{L}_5 = J_R\cdot 2\pi\mathcal{R}_5 . \tag{5.12}$$

Again the rescaling is reminiscent of the "long string" rescaling (4.15) of BPS black holes.

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[9] Those corresponds to a more refined description where one Frolov-Thorne temperature for each angle is introduced $\beta_{{\rm FT}\phi}, \beta_{{\rm FT}\psi}$. This gives rise to two independent central charges, one for each component of the angular momentum $J_{\phi,\psi}$. It is the sum of those that give (5.11).

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